

# Asymptotic Structure and Symmetries of the Open FLRW Universe

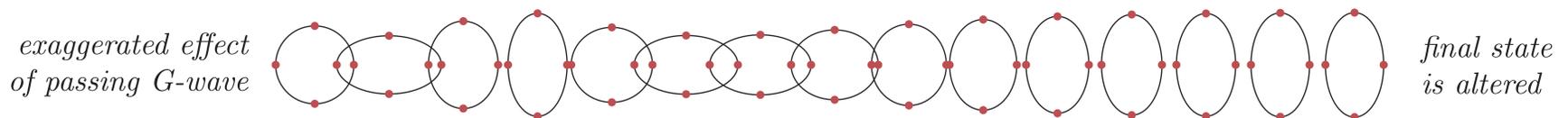


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## Introduction · Gravitational Waves, Memory, and Asymptotic Symmetries

Spacetime is a dynamical entity coupled to energy–momentum. **Gravitational waves** are perturbations in spacetime propagating at the speed of light caused by accelerated masses. Gravitational waves carry energy (causing the in-spiral of binary systems) and, after their passage, permanently displace free-falling particles relative to one another—a phenomenon known as **gravitational memory**. The memory effect may be detectable by mid-2030 with next-generation instruments such as the Laser Interferometer Space Antenna (LISA).



**Asymptotic symmetries** are the symmetries of *null infinity*  $\mathcal{I}$ —the region approached by following light paths asymptotically far from sources. An elegant formulation of gravitational memory arises in terms of these asymptotic symmetries...

## I · Open FLRW Universes

We investigated the asymptotic structure of Friedmann–Lemaître–Robertson–Walker (FLRW) universes with negative (open) spatial curvature. The expanding, dust-filled open FLRW universe is described by

$$ds^2 \propto (\cosh \eta - 1)^2 \left[ -d\eta^2 + \frac{dr^2}{1+r^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

which becomes flat to the far future as  $\eta \rightarrow \infty$ . Surprisingly, we found that the open FLRW universe fails the Bondi–Sachs criterion [1] of asymptotic flatness, even as  $\eta \rightarrow \infty$ . However, we found the open FLRW universe was manifestly asymptotically future flat by considering the deviation of a beam of parallel light rays of area  $A$  in the far future limit.

$$\begin{array}{l} \text{geodesic} \\ \text{deviation} \\ \text{equation} \end{array} \implies \underbrace{\frac{1}{\sqrt{A}} \frac{d^2}{d\lambda^2} \sqrt{A}}_{\text{rate of divergence}} \propto \underbrace{\left( \frac{dt}{d\lambda} \right) \frac{1}{t^3}}_{\text{vanishes as } \eta \rightarrow \infty} \rightarrow 0$$

## III · Asymptotic Symmetries

Symmetries of a spacetime are *isometries* generated as the flow of vector fields  $\zeta$  which satisfy Killing's equation

$$\mathcal{L}_\zeta g_{ab} = 0.$$

E.g., the symmetries of Minkowski spacetime  $g_{ab} = \eta_{ab}$  form the Poincaré group. Asymptotic symmetries are described by a relaxation of Killing's equation

$$\mathcal{L}_\zeta g_{ab} = \mathcal{O}(\delta g_{ab}),$$

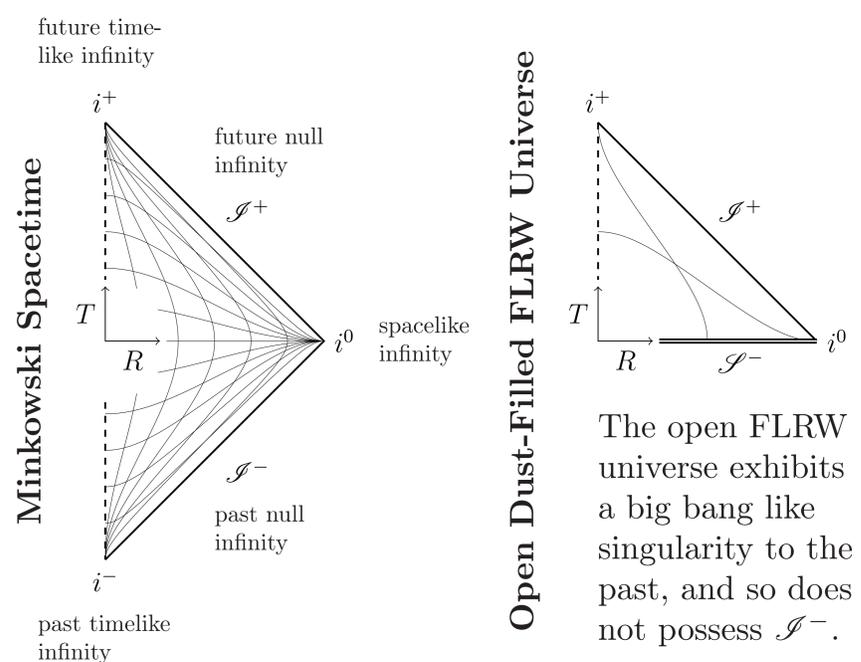
where  $\delta g_{ab}$  are permitted perturbations which falloff near  $\mathcal{I}$  at a prescribed rate. There is no canonical choice of falloff rate. The natural choice of BMS [1,2] yields the asymptotic symmetries generated by

$$\zeta = f \partial_t + \mathcal{D}^2 f \partial_r - \frac{1}{r} \mathcal{D}^A f \partial_A \quad \begin{cases} x^A : \text{angular coordinates} \\ \mathcal{D}^A : \text{derivative on 2-sphere} \end{cases}$$

where  $f = f(x^A)$  is any function of the 2-sphere. Together, these symmetries form the *BMS group*.

## II · Asymptotic Structure

The asymptotic structure of the open FLRW universe was characterised via *conformal compactification*, wherein the infinite spacetime is mapped onto the interior of a finite region with a boundary.



The open FLRW universe exhibits a big bang like singularity to the past, and so does not possess  $\mathcal{I}^-$ .

## IV · BMS Supertranslations

Surprisingly, the *asymptotic symmetries* of Minkowski space (the BMS group) is a superset of the *interior symmetries* of Minkowski space (the Poincaré group). Normal spacetime translations are recovered when  $f$  is a  $\ell \in \{0, 1\}$  spherical harmonic, but higher harmonics generate an infinitude of “supertranslations” not present in the Poincaré group.

Gravitational memory is a consequence of the fact that gravitational waves induce transitions among *physically distinct vacua*, and these vacua are related by supertranslations.

We endeavour to generalise the BMS analysis to open FLRW universes, but haven't been able to yet.

- [1] H Bondi, M Gr J Van der Burg, and A. Metzner. Waves from axisymmetric isolated system. *Proc. R. Soc. Lond. Series A. Math. & Phys.*, 269(1336), 1962.
- [2] R. K. Sachs. Asymptotic symmetries in gravitational theory. *Phys. Rev.*, 128:2851–2864, 1962.
- [3] A. Ashtekar, Miguel Campiglia, and Alok Laddha. Null infinity, the BMS group and infrared issues. *General Relativity and Gravitation*, 50(11):140, 2018.