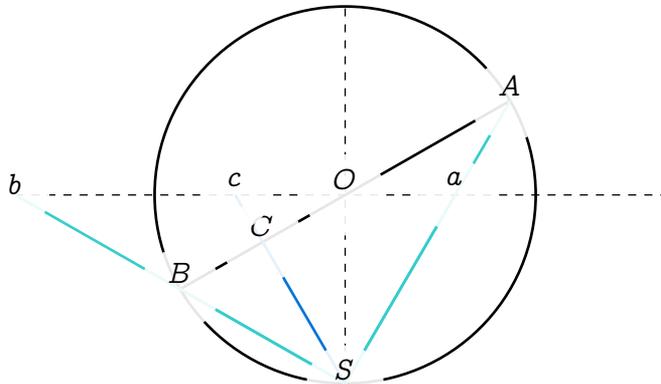


Midpoint of stereographic projection of antipedal points

Theorem. Let A and B be opposite points on a circle, and let a and b the stereographic projection of A and B into the x -axis through the south pole S . Then the midpoint a and b is the point c where the perpendicular to AB through the south pole meets the x -axis.



Proof. Because the line SC is perpendicular to the hypotenuse AB , it cuts the right triangle ABS into two other right triangles ASC and SBC . All three triangles are similar. Similarly, the line OS splits the right triangle baS into SaO and bsO , all of which are similar. Furthermore, since AOS is isosceles, triangles ABS and SaO share angles and so are also similar. Both groups of similar triangles are therefore similar to each other, and in particular, triangles bsO and SBC are similar, meaning bcS is isosceles. The two isosceles triangles share a leg, so $bc = Sc = ac$ and hence c is the midpoint of ab .