

# Summary of CGA blade IPNS and OPNS

[See here for the derivation.](#)

$X$	opns	ipns
$\mathbf{E}\infty$	$\{\infty\}$	$\{\infty\}$
$\mathbb{T}_p[\mathbf{E}]$	$\emptyset$	$\{\text{up}(p + u) \mid u \perp \mathbf{E} = 0\} \cup \{\infty\}$
$\mathbb{T}_p[\sigma \wedge \mathbf{E} \wedge \infty]$	$\{\text{up}(p + u) \mid u \wedge \mathbf{E} = 0\} \cup \{\infty\}$	$\emptyset$
$\mathbb{T}_p[(\sigma + \frac{1}{2}r^2\infty) \wedge \mathbf{E}]$	$\{\text{up}(p + u) \mid u \wedge \mathbf{E} = 0, u^2 = r^2\}$	$\{\text{up}(p + u) \mid u \perp \mathbf{E} = 0, u^2 = -r^2\}$
$\mathbb{T}_p[(\sigma - \frac{1}{2}r^2\infty) \wedge \mathbf{E}]$	$\{\text{up}(p + u) \mid u \wedge \mathbf{E} = 0, u^2 = -r^2\}$	$\{\text{up}(p + u) \mid u \perp \mathbf{E} = 0, u^2 = +r^2\}$
$\mathbb{T}_p[\sigma \wedge \mathbf{E}]$	$\{\text{up}(p)\}$	$\{\text{up}(p)\}$

If we assume the base space is Euclidean, so that  $u^2 > 0$  and  $u^2 = 0 \iff 0$ , then the table simplifies slightly:

$X$	opns	ipns
$\mathbf{E}\infty$	$\{\infty\}$	$\{\infty\}$
$\mathbb{T}_p[\mathbf{E}]$	$\emptyset$	$\{\text{up}(p + u) \mid u \perp \mathbf{E} = 0\} \cup \{\infty\}$
$\mathbb{T}_p[\sigma \wedge \mathbf{E} \wedge \infty]$	$\{\text{up}(p + u) \mid u \wedge \mathbf{E} = 0\} \cup \{\infty\}$	$\emptyset$
$\mathbb{T}_p[(\sigma + \frac{1}{2}r^2\infty) \wedge \mathbf{E}]$	$\{\text{up}(p + u) \mid u \wedge \mathbf{E} = 0, u^2 = r^2\}$	$\emptyset$
$\mathbb{T}_p[(\sigma - \frac{1}{2}r^2\infty) \wedge \mathbf{E}]$	$\emptyset$	$\{\text{up}(p + u) \mid u \perp \mathbf{E} = 0, u^2 = +r^2\}$
$\mathbb{T}_p[\sigma \wedge \mathbf{E}]$	$\{\text{up}(p)\}$	$\{\text{up}(p)\}$