Proof that $\exp([A,-])X = \exp(A)X\exp(-A)$.

Lemma 1

$$\exp([A,-])X = \exp(A)X\exp(-A)$$

Proof Expanding the r.h.s.,

$$\begin{aligned}
\exp(A)X\exp(-A) &= \left[\sum_{n=0}^{\infty}\frac{1}{n!}A^n\right]X\left[\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}A^n\right] \\
&= \sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(-1)^k}{k!(n-k)!}A^{n-k}XA^k \\
&= \sum_{n=0}^{\infty}\frac{1}{n!}[A,-]^n X \quad \text{via Lemma 2} \\
&= \exp([A,-])X
\end{aligned}$$

Lemma 2

$$[A,-]^n X = \sum_{k=0}^{n}(-1)^k\binom{n}{k}A^{n-k}XA^k$$

Proof by induction. Assuming true for n,

$$[A,-]^n AX = \sum_{k=0}^{n+1}(-1)^k\binom{n+1}{k}\frac{n+1-k}{n+1}A^{(n+1)-k}XA^k$$

via Lemma 3, and

$$\begin{aligned}
[A,-]^n XA &= \sum_{k=0}^{n}(-1)^k\binom{n+1}{k+1}\frac{k+1}{n+1}A^{(n+1)-(k+1)}XA^{k+1} \\
&= -\sum_{\bar{k}=0}^{n+1}(-1)^{\bar{k}}\binom{n+1}{\bar{k}}\frac{\bar{k}}{n+1}A^{(n+1)-\bar{k}}XA^{\bar{k}}
\end{aligned}$$

via Lemma 4. Taking both together,

$$\begin{aligned}
[A,-]^{n+1}X &= [A,-](AX - XA) \\
&= \sum_{k=0}^{n+1}(-1)^k\binom{n+1}{k}A^{(n+1)-k}XA^k
\end{aligned}$$

which shows that the $n+1$ case holds, and hence $\forall n$.

Lemma 3

$$\binom{n}{k} = \binom{n+1}{k}\frac{n+1-k}{n+1}$$

Proof

$$\frac{n!}{k!(n-k)!} = \frac{(n+1)!}{k!(n+1-k)!}\frac{n+1-k}{n+1}$$

Lemma 4

$$\binom{n}{k} = \binom{n+1}{k+1}\frac{k+1}{n+1}$$

Proof

$$\frac{n!}{k!(n-k)!} = \frac{(n+1)!}{(k+1)!(n-k)!}\frac{k+1}{n+1}$$