Compactification
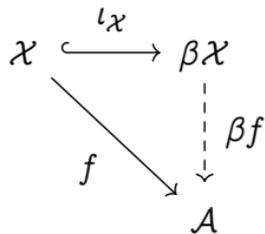
A compactification of $\mathcal{X}$ is an embedding $\phi : \mathcal{X} \to \mathcal{Z}$ where $\phi(\mathcal{X})$ is a dense subset of the compact space $\mathcal{Z}$.

Stone–Čech compactification

The Stone–Čech compactification $\beta\mathcal{X}$ of a topological space $\mathcal{X}$ is the most general compact Hausdorff space.

$$\mathcal{X} \xhookrightarrow{\iota_\mathcal{X}} \beta\mathcal{X}$$

Any continous map $f : \mathcal{X} \to \mathcal{A}$ where $\mathcal{A}$ is compact Hausdorff factors **uniquely** as $f = \beta f \circ \iota_\mathcal{X}$.

Construction