Dypin Cyclides

See [dupin-cyclide-3d-plot].

Parametric form

$$
\begin{aligned}
x &= \frac{d(c - a\cos u\cos v) + b^2\cos u}{m} \\
y &= \frac{b\sin u(a - d\cos v)}{m} \\
z &= \frac{b\sin v(c\cos u - d)}{m} \\
m &= a - c\cos u\cos v \\
a &= \sqrt{b^2 + c^2} \\
u, v &\in [0, \tau)
\end{aligned}
$$

Interpretations

Given b, c, d:

- b is the major radius
- c is related to eccentricity; $c = 0$ is a circle
- d is the average thickness

The radius of a channelling sphere is $r = \frac{c}{b}x$

Lie Sphere Geometry

With [lie-sphere-geometry], a Dupin cyclide is the OPNS of:

$$
\mathsf{T}_{te_0}\left[\left(n_0 + \frac{1}{2}\rho^2 n_\infty\right) \wedge (\lambda e_1 + e_0) \wedge e_2\right]
$$

Parametric form	OPNS form
b	ρ
c	$1/\sqrt{\lambda^2 - 1}$
$\sqrt{1 + 1/c^2}$	λ
d	t

$$
m = \frac{c}{b} = \frac{1}{\sqrt{\lambda^2 - 1}}
$$