Charles Gunn's master's thesis (Gunn, 2011)

§2.2.1) Defines determinant of a linear map $f : V \to V$ as $\Delta(\{\boldsymbol{v}_i\}) \coloneqq \alpha$ such that the exterior power $f(\boldsymbol{v}_1) \wedge \cdots \wedge f(\boldsymbol{v}_n) = \alpha \boldsymbol{I}$ but does not require that $\boldsymbol{I} = \boldsymbol{v}_1 \wedge \cdots \wedge \boldsymbol{v}_n$.

Uses this to establish a **canonical isomorphism** between V and $\bigwedge^{n-1} V^*$.

§2.2.3.1) Defines the adjoint of a linear map $f : V \to V^*$ by forming the $(n-1)$th exterior power $\bigwedge^{n-1} f : \bigwedge^{n-1} V \to \bigwedge^{n-1} V^*$ and using the canonical isomorphism above to map this to $f : V^* \to V$.

This is interesting, because it is a **metric independent definition of the adjoint**.

§2.3) Introduces the Poincaré isomorphism $J : \mathbb{P}(\bigwedge V) \to \mathbb{P}(\bigwedge V^*)$ and seems to imply that it is canonical and metric independent, but doesn't say or show this.

Surely there are many such isomorphisms? Why this one?

References

Gunn, C. (2011). On the Homogeneous Model of Euclidean Geometry. In L. Dorst & J. Lasenby (Eds.), *Guide to Geometric Algebra in Practice: Guide to Geometric Algebra in Practice* (pp. 297–327). Springer. https://doi.org/10.1007/978-0-85729-811-9_15