Identities involving products in geometric algebra

In the formulae below, let u be a 1-vector and let $\{A, B, C\}$ be general multivectors.

Vector products (Wilson, 2022, lemma 10):

$$u \rfloor A = \frac{1}{2}(uA - A^\star u)$$

$$u \wedge A = \frac{1}{2}(uA + A^\star u)$$

Vector contraction anti-derivations (Wilson, 2022, corollary 1):

$$u \rfloor (AB) = (u \rfloor A)B + A^\star (u \rfloor B)$$

$$u \rfloor (A \wedge B) = (u \rfloor A) \wedge B + A^\star \wedge (u \rfloor B)$$

Double contractions (Wilson, 2022, lemma 14):

$$(A \lfloor B) \lfloor C = A \lfloor (B \wedge C)$$

$$A \rfloor (B \rfloor C) = (A \wedge B) \rfloor C$$

Contraction associativity (Wilson, 2022, lemma 15):

$$(A \rfloor B) \lfloor C = A \rfloor (B \lfloor C)$$

(Hestenes & Sobczyk, 1984, equations 1.41)

$$u \rfloor (AB) = (u \rfloor A)B + A^\star (u \rfloor B) = (u \wedge A)B - A^\star (u \wedge B)$$

$$u \wedge (AB) = (u \rfloor A)B + A^\star (u \wedge B) = (u \wedge A)B - A^\star (u \rfloor B)$$

References

Hestenes, D., & Sobczyk, G. (1984). *Clifford Algebra to Geometric Calculus*. Springer Netherlands. https://doi.org/10.1007/978-94-009-6292-7

Wilson, J. (2022). *Geometric Algebra for Special Relativity and Manifold Geometry*. https://doi.org/10.26686/wgtn.21185911