

Multivariate Gaussian distributions

The D -dimensional Gaussian pdf with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ is

$$\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) := \frac{1}{\sqrt{\tau^D \det(\boldsymbol{\Sigma})}} \exp\left(-\frac{1}{2}(x - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(x - \boldsymbol{\mu})\right)$$

A Gaussian's parameters are fully specified by the coefficients of x in the exponent.

$$\begin{aligned} -2 \ln \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) &= (x - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(x - \boldsymbol{\mu}) + \text{constant} \\ &= x^T \boldsymbol{\Sigma}^{-1} x - x^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} - \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} x + \text{constant} \end{aligned}$$