

Lattice structure for \mathbb{O}/\mathbb{I}

join	$\mathbb{O}(B)$	$\mathbb{I}(B)$
$\mathbb{O}(A)$	$\mathbb{O}(A \wedge B) = \mathbb{I}(A^* \vee B^*)$	$\mathbb{O}(A \wedge B^*) = \mathbb{I}(A \mid B)$
$\mathbb{I}(A)$	$\mathbb{O}(A^* \wedge B) = \mathbb{I}(A \lfloor B)$	$\mathbb{O}(A^* \wedge B^*) = \mathbb{I}(A \vee B)$
meet	$\mathbb{O}(B)$	$\mathbb{I}(B)$
$\mathbb{O}(A)$	$\mathbb{O}(A \vee B) = \mathbb{I}(A^* \wedge B^*)$	$\mathbb{O}(A \lfloor B) = \mathbb{I}(A^* \wedge B)$
$\mathbb{I}(A)$	$\mathbb{O}(A \mid B) = \mathbb{I}(A \wedge B^*)$	$\mathbb{O}(A^* \vee B^*) = \mathbb{I}(A \wedge B)$

Without explicit duals

join	$\mathbb{O}(B)$	$\mathbb{I}(B)$
$\mathbb{O}(A)$	$\mathbb{O}(A \wedge B)$	$\mathbb{I}(A \mid B)$
$\mathbb{I}(A)$	$\mathbb{I}(A \lfloor B)$	$\mathbb{I}(A \vee B)$
meet	$\mathbb{O}(B)$	$\mathbb{I}(B)$
$\mathbb{O}(A)$	$\mathbb{O}(A \vee B)$	$\mathbb{O}(A \lfloor B)$
$\mathbb{I}(A)$	$\mathbb{O}(A \mid B)$	$\mathbb{I}(A \wedge B)$

Proof.

$$\mathbb{O}(A) \text{ join } \mathbb{I}(B) = \mathbb{O}(A) \text{ join } \mathbb{O}(B^*) = \mathbb{O}(A \wedge B^*) = \mathbb{I}(A \mid B)$$

$$\mathbb{I}(A) \text{ meet } \mathbb{O}(B) = \mathbb{I}(A) \text{ meet } \mathbb{I}(B^*) = \mathbb{I}(A \wedge B^*) = \mathbb{O}(A \mid B)$$