Topology of projective spaces

Define the **real projective space**

$$\mathbb{R}P^n := \left(\mathbb{R}^{n+1} \setminus \{0\}\right) / \sim$$

where $u \sim v \iff u = \lambda v$ for some nonzero $\lambda \in \mathbb{R}$.

Define the **Grassmannian** $\mathrm{Gr}_k(V)$ as the space of k-dimensional linear subspaces of V.