Rubik's cube implementation

First, we give the six cubic directions names, x, y and z, forming a right-hand frame, and X, Y and Z, which are the opposites. The faces of the standard Rubik's cube in each direction are coloured by their directions.

Next, we define how the directions twist each other. Below is a tabulation of the direction obtained by rotating the horizontal direction 90° clockwise the vertical another direction as seen looking inwards from that direction.

	x	y	z	X	Y	Z
x	x	Z	y	X	z	Y
y	z	y	X	Z	Y	x
z	Y	x	z	y	X	Z
X	x	z	Y	X	Z	y
Y	Z	y	x	z	Y	X
Z	y	X	z	Y	x	Z

For example, rotating x around y clockwise gives z.

Sticker addressing

We identify stickers as tuples of directions. The six center pieces are specified with their direction (e.g., z is the blue sticker on the front face), edge pieces are specified with the face followed by the side of the face (e.g., zx is the blue-red piece on the right side of the front face) and corner pieces similarly by the face followed by the two directions toward the corner.

There are two valid ways of addressing each corner piece (e.g., zyx and zxy both identify the top-right piece on the front face) so we define a sticker-address normalisation function which always picks one.

The state of the cube is represented as an associative map from sticker addresses to sticker colours.

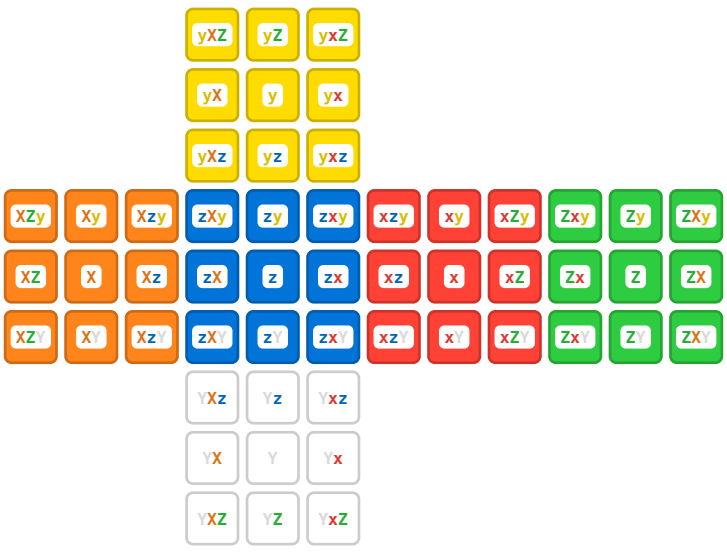

Figure 1: Solved state showing sticker addresses.

Figure 2: Solved state showing sticker addresses.

Twisting stickers

Stickers can be permuted by twisting each direction in their address. For example, rotating sticker zyx around the x direction gives yZx

Defining moves

Identity $\mathbb{I} =$ 

Cube rotations $\mathrm{x} =$  , $\mathrm{y} =$  , $\mathrm{z} =$ 

Twists $\mathrm{R} =$  , $\mathrm{L} =$  , $\mathrm{U} =$  , $\mathrm{D} =$  , $\mathrm{F} =$  , $\mathrm{B} =$ 

Inverse twists $\mathrm{R}' =$  , $\mathrm{L}' =$  , $\mathrm{U}' =$  , $\mathrm{D}' =$  , $\mathrm{F}' =$  , $\mathrm{B}' =$ 

Double twists $\mathrm{R}^2 =$  , $\mathrm{L}^2 =$  , $\mathrm{U}^2 =$  , $\mathrm{D}^2 =$  , $\mathrm{F}^2 =$  , $\mathrm{B}^2 =$ 

Wide twists $\mathrm{r} =$  , $\mathrm{l} =$  , $\mathrm{u} =$  , $\mathrm{d} =$  , $\mathrm{f} =$  , $\mathrm{b} =$ 

Inverse wide twists $\mathrm{r}' =$  , $\mathrm{l}' =$  , $\mathrm{u}' =$  , $\mathrm{d}' =$  , $\mathrm{f}' =$  , $\mathrm{b}' =$ 

Double wide twists $\mathrm{r}^2 =$  , $\mathrm{l}^2 =$  , $\mathrm{u}^2 =$  , $\mathrm{d}^2 =$  , $\mathrm{f}^2 =$, $\mathrm{b}^2 =$

Slice twists $\mathrm{M} =$, $\mathrm{E} =$, $\mathrm{S} =$

Inverse slice twists $\mathrm{M}' =$, $\mathrm{E}' =$, $\mathrm{S}' =$

Double slice twists $\mathrm{M}^2 =$, $\mathrm{E}^2 =$, $\mathrm{S}^2 =$

Algebraic operations

Conjugate $\langle X : Y \rangle = X\,Y\,X'$

Commutator $[X, Y] = X\,Y\,X'\,Y'$

Reflections $\mathtt{reflect}(\mathrm{x}, \mathrm{R\,L\,U\,D\,F\,B}) = \mathrm{L'\,R'\,U'\,D'\,F'\,B'}$

Rotations $\mathtt{rotate}(\mathrm{x}, \mathrm{R\,L\,U\,D\,F\,B}) = \mathrm{R\,L\,B\,F\,U\,D}$