The spin groups

Define. Let V be a vector space with quadratic form q. Let $\mathbb{S} := \{u \in V \mid q(u) = \pm 1\}$ be the generalised unit sphere.

$$\begin{aligned} \mathrm{Pin}(V, q) &:= \{u_1 u_2 \cdots u_k \mid u_i \in \mathbb{S}\} \\ \mathrm{Spin}(V, q) &:= \{u_1 u_2 \cdots u_{2k} \mid u_i \in \mathbb{S}\} \\ &= \mathrm{Pin}(V, q) \cap Cl(V, q)^+ \end{aligned}$$

Nit-picking definitions

In (Lawson & Michelsohn, 1989, definition 2.3), the following subgroups of $Cl(V, q)$ are defined:

- $Cl^\times(V, q) = \left\{\varphi \in Cl(V, q) \mid \exists \varphi^{-1}\right\}$, or just invertible elements
- $\mathrm{P}(V, q)$ is the subgroup of $Cl^\times(V, q)$ generated by vectors $v \in V$ with $v^2 \neq 0$
- $\mathrm{Pin}(V, q)$ is the subgroup of $\mathrm{P}(V, q)$ generated by vectors $v \in V$ with $v^2 \neq \pm 1$
- Finally, $\mathrm{Spin}(V, q)$ is the even subgroup $\mathrm{Pin}(V, q) \cap Cl^0(V, q)$

Later on, (Lawson & Michelsohn, 1989, equation 2.24) states the pin group is

$$\mathrm{Pin}(V, q) = \left\{u_1 u_2 \cdots u_k \in \mathrm{P}(V, q) \;\middle|\; u_i^2 = \pm 1\right\}.$$

Why bother with defining $\mathrm{P}(V, q)$, and $Cl^\times(V, q)$, for that matter? Is it true that:

$$\mathrm{Pin}(V, q) = \left\{u_1 u_2 \cdots u_k \;\middle|\; u_i^2 = \pm 1\right\} \ ?$$

That would be cleaner.

I'm going to say "yes".

References

Lawson, H. B., & Michelsohn, M.-L. (1989). *Spin Geometry* (Issue 38). Princeton University Press.